

On the role of hyperfine-interactions-mediated Zeeman effect in the condensation temperature shift of trapped atomic Bose-Einstein condensates

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We discuss the effect of interatomic interactions on the condensation temperature T_c of a laboratory atomic Bose-Einstein condensate under the influence of an external trapping magnetic field. We predict that accounting for hyperfine interactions mediated Zeeman term in the mean-field approximation produces, in the case of the 403 G Feshbach resonance in the $|F, m_F\rangle = |1, 1\rangle$ hyperfine state of a ^{39}K condensate, with F the total spin of the atom, an experimentally observed (and not yet explained) shift in the condensation temperature $\Delta T_c/T_c^0 = b_0^* + b_1^*(a/\lambda_T) + b_2^*(a/\lambda_T)^2$ with $b_0^* \simeq 0.0002$, $b_1^* \simeq -3.4$ and $b_2^* \simeq 47$, where a is the s-wave scattering length, and λ_T is the thermal wavelength at T_c^0 . Generic expressions for the coefficients b_0^* , b_1^* and b_2^* are also obtained, which can be used to predict the temperature shift for other Feshbach resonances of ^{39}K or other atomic condensates.

PACS numbers: 67.85.Hj, 67.85.Jk

The study of Bose-Einstein condensates (BECs) is a important current subject of modern physics (see Refs[1–3] for a review). Atomic BECs are produced in the laboratory in laser-cooled, magnetically-trapped ultra-cold bosonic clouds of different atomic species, e.g., ^{87}Rb [4], ^7Li [5], ^{23}Na [6], ^1H [7], ^{87}Rb [8], ^4He [9], ^{41}K [10], ^{133}Cs [11], ^{174}Yb [12] and ^{52}Cr [13]. Also, BECs of photons are nowadays under investigation [14]. Moreover, BECs are commonly applied in cosmology and astrophysics [15] and in fact have been shown also to constrain quantum gravity models [16].

In the context of atomic BECs, inter-particle interactions play a fundamental role, since they are necessary to drive the atomic cloud to thermal equilibrium, so they must carefully be taken into account when studying the properties of the condensate. For instance, interatomic interactions change the condensation temperature T_c of a BEC, as was pointed out first by Lee and Yang [17, 18] (see also [19–28] for more recent works).

The first studies of interactions effects were focused on uniform BECs. Here, interactions are absent in the mean field (MF) approximation (see [23, 26–28] for reviews) but they produce a shift in the condensation temperature with respect to the ideal noninteracting case, which is due to beyond-MF effects related to quantum correlations between bosons near the critical point. This effect has been finally quantified in [23, 24] as $\Delta T_c/T_c^0 \simeq 1.8(a/\lambda_T)$, where $\Delta T_c \equiv T_c - T_c^0$ with T_c the critical temperature of the gas of interacting bosons, T_c^0 the condensation temperature in the ideal non-interacting case, $\lambda \equiv \sqrt{2\pi\hbar^2/(m_a k_B T_c^0)}$ the thermal wavelength at temperature T_c^0 , m_a the atom mass, and a the s-wave scattering length used to parameterize inter-particle interactions [1–3].

It should be noted that laboratory condensates are not uniform BECs since they are produced in atomic clouds confined in magnetic traps, but they can be described in terms of harmonically-trapped BECs consisting of a system of N bosons trapped in an external spherically symmetric harmonic potential $V = m_a \omega^2 x^2/2$, with ω the frequency of the trap. For trapped BECs, interactions affect the condensation temperature even in the MF approximation, and the shift in T_c in terms of the s-wave scattering length is given by

$$\Delta T_c/T_c^0 \simeq b_1(a/\lambda_T) + b_2(a/\lambda_T)^2. \quad (1)$$

with $b_1 \simeq -3.426$ [1] and $b_2 \simeq 18.8$ [29], implying that ΔT_c is negative for repulsive interactions [43].

High precision measurements [36] of the condensation temperature of ^{39}K in the range of parameters $N \simeq (2 - 8) \times 10^5$, $\omega \simeq (75 - 85)\text{Hz}$, $10^{-3} < a/\lambda_T < 6 \times 10^{-2}$ and $T_c \simeq (180 - 330)\text{nK}$ have detected second-order effects in $\Delta T_c/T_c^0$ fitted by the following expression $\Delta T_c/T_c^0 \approx b_1^{exp}(a/\lambda_T) + b_2^{exp}(a/\lambda_T)^2$ with $b_1^{exp} \simeq -3.5 \pm 0.3$ and $b_2^{exp} \simeq 46 \pm 5$. This result has been achieved exploiting the 403 G Feshbach resonance in the $|F, m_F\rangle = |1, 1\rangle$ hyperfine state of a ^{39}K condensate, where F is the total (nuclear + electronic) spin of the atom. Therefore, if the theoretically predicted linear contribution b_1 was found to practically coincide with the observed value, its nonlinear (quadratic) counterpart b_2 turned out to be in strong disagreement with experimental data.

It should be mentioned that there have been some efforts to theoretically estimate the correct value of b_2 in the MF approximation, for instance considering anharmonic and even temperature-dependent traps [30], which however have been not successful. Therefore one could

expect that a more realistic prediction of the experimental value of b_2^{exp} should take into account beyond-MF effects.

The goal of this paper is to show that, taking into account the Zeeman effect and using the MF approximation, it is quite possible to predict the experimentally observed value of $b_2 \simeq 47$ for the 403G resonance of the hyperfine $|F, m_F\rangle = |1, 1, \rangle$ state of ^{39}K as measured in [36], with no need to appeal to any beyond-MF effects. We also discuss the generality of this result and the possibility of predicting the condensation temperature shift for different resonances of ^{39}K and for different atomic condensates.

Recall that in the MF Hartree-Fock approximation (assuming the semiclassical condition $k_B T \gg \hbar\omega$), bosons are treated as a noninteracting gas that experiences a MF interaction potential $\propto gn(x)$, where $g = (4\pi\hbar^2 a/m_a)$ [1–3] and $n(x)$ is the local density of bosons at the point x , so that the Hartree-Fock hamiltonian is [1–3]

$$H_{HF} = \frac{P^2}{2m_a} + V(x) + 2gn(x) \quad (2)$$

It is important to remind that experimentally the s-wave scattering length parameter a is tuned via the Feshbach-resonance technique based on Zeeman splitting of bosonic atom levels in applied magnetic field. It means that g in Eq.(2) is actually *always* field-dependent, since $g \propto a = a(B)$. More explicitly, according to the interpretation of the Feshbach resonance [31, 32]

$$a(B) = a_0 \left(1 - \frac{\Delta}{B - B_0} \right) \quad (3)$$

where a_0 is the so-called background value of the length, B_0 is the resonance peak field, and Δ the width of the resonance.

Thus, in order to properly address the problem of condensation temperature shifting (which is always observed under application of a non-zero magnetic field B), one has to add to Eq. (2) a missing hyperfine interactions mediated Zeeman contribution $H_Z = -\mu(x)B$ where $\mu(x)$ is the local magnetic moment of a Bose atom in the trap. Since an applied magnetic field affects the condensate, this moment depends on the local density $n(x)$ as follows, $\mu(x) = \mu_a n(x) V_m$ where $\mu_a = g_s S \mu_N$ is the magnetic moment of a particular atom with S the nuclear spin, g_s the gyromagnetic coefficient, and $\mu_N = e\hbar/2m_p$ the nuclear magneton (m_p being the proton mass). Here $V_m = 4\pi a_m^3$ is a characteristic volume of the condensate affected by hyperfine interactions between atoms with a_m being a magnetic analog of the scattering length a . According to the spectroscopic data [31], there are singlet (a_S) and triplet (a_T) scattering lengths.

It can be easily verified that accounting for the Zeeman contribution in Eq.(2) will result in a simple renormalization of the interaction constant $g(B)$ (which depends on

applied magnetic field via the s-wave length $a(B)$ given by Eq.(3)) as follows

$$g^*(B) = g(B) - \frac{1}{2}\mu_a B V_m \quad (4)$$

and the corresponding scattering length

$$a^*(B) = a(B) - \alpha B \quad (5)$$

with $\alpha = \mu_a m_a V_m / 8\pi\hbar^2$.

Now by inverting Eq.(3) and expanding the resulting $B(a)$ dependence into the Taylor series, one obtains

$$a^* = a - \alpha B_0 \left(1 + \frac{\Delta}{B_0} \right) - \alpha \Delta \left[\frac{a}{a_0} + \left(\frac{a}{a_0} \right)^2 + \dots \right] \quad (6)$$

for an explicit form of the renormalized (due to Zeeman splitting) scattering length $a^*(B)$. Now, to find the changes of the amplitudes b_1 and b_2 in the presence of the Zeeman effect, we simply replace the original (Zeeman-free) scattering lengths a in Eq.(1) with their renormalized counterparts a^* (given by Eq. (6)) which will result in the following expression for the temperature shift

$$\frac{\Delta T_c}{T_c^0} = b_1 \left(\frac{a^*}{\lambda_T} \right) + b_2 \left(\frac{a^*}{\lambda_T} \right)^2 \quad (7)$$

Now, by using Eq.(6) we can rewrite Eq.(7) in terms of the original scattering lengths a and renormalized amplitudes b_i^* as follows

$$\frac{\Delta T_c}{T_c^0} = b_0^* + b_1^* \left(\frac{a}{\lambda_T} \right) + b_2^* \left(\frac{a}{\lambda_T} \right)^2 \quad (8)$$

where the new amplitudes (due to the Zeeman contribution) are governed by the following expressions

$$b_0^* = - \left(\frac{\xi}{\lambda_T} \right) b_1 + \left(\frac{\xi}{\lambda_T} \right)^2 b_2, \quad (9)$$

$$b_1^* = (1 - \gamma)b_1 - 2(1 - \gamma) \left(\frac{\xi}{\lambda_T} \right) b_2, \quad (10)$$

and

$$b_2^* = \left[(1 - \gamma)^2 + 2\gamma \left(\frac{\xi}{a_0} \right) \right] b_2 - \gamma \left(\frac{\lambda_T}{a_0} \right) b_1 \quad (11)$$

where $\gamma = \alpha\Delta/a_0$, and $\xi = \alpha(B_0 + \Delta)$.

Note that accounting for Zeeman effect resulted in the appearance of a constant amplitude b_0^* . As we shall demonstrate below, this contribution is very small and

does not affect the experimentally observed temperature shift.

To fix the model parameters, we proceed as follows. First of all, we quite reasonably assume that Zeeman effect does not change the linear contribution by putting $b_1^* = b_1$. Secondly, to find the absolute change of the second amplitude due to Zeeman term, we assume that $b_2^* = cb_2$ where c is a constant (amplifying factor). In view of Eqs.(9-11), the above two assumptions bring about the following analytical expression for the seeking amplifying parameter

$$c = 1 + \frac{2g_s S \mu_N m_a B_0 a_m^3}{\hbar^2 a_0} = 1 + S \left(\frac{B_0}{B_m} \right) \quad (12)$$

which is the main result of this paper. To obtain the second form of the above expression, we have introduced a characteristic magnetic field B_m related to hyperfine interactions. More precisely, $B_m = \Phi_0/\sigma_m$ where $\Phi_0 = h/2e = 2 \times 10^{-15} \text{ Wb}$ is the flux quantum and the projected area σ_m is given by $\sigma_m = \pi g_s (m_a/m_p) a_m^3/a_0$. Note that, as expected, in the absence of Zeeman effect (when $\mu_a = 0$), we have $c = 1$ and thus $b_2^* = b_2$.

Let us consider the particular case of the 403G resonance of the hyperfine $|F, m_F\rangle = |1, 1\rangle$ state of ^{39}K . According to [31], the relevant parameters needed to create and measure magnetically trapped bosons for this atom are as follows: $S = 3/2$, $g_s = 1/2$, $m_a = 39m_p$, $B_0 = 403\text{G}$, $a_0 = -29r_B$ (where $r_B = 0.053\text{nm}$ is the Bohr radius), and $a_S = 138r_B$ (for the singlet magnetic scattering length). According to our Eq.(12), the above

set of parameters produces $c \simeq 2.5$ which readily leads to the following estimate of the quadratic amplitude contribution due to the Zeeman effect, $b_2^* = 2.5b_2 \simeq 47$ in a good agreement with observations[36]. It is interesting to point out that since the nuclear spin of ^{39}K is $S = 3/2$, the obtained value $c \simeq 2.5$ for the amplifying factor means that we have practically a complete match between the Feshbach resonance field B_0 and the hyperfine interaction related field B_m , namely $B_m \simeq B_0$.

To check self-consistency of our calculations, we also estimated the value of the constant amplitude b_0^* (which is equal to zero in the absence of the Zeeman effect). The result is $b_0^* \simeq 0.0002$ which is, as expected, a rather negligible contribution, even though it is not zero.

In conclusion, we have shown that accounting for hyperfine-interactions-induced Zeeman term in the mean-field approximation produces, for the 403G Feshbach resonance in the $|F, m_F\rangle = |1, 1\rangle$ hyperfine state of a ^{39}K condensate, an experimentally observed shift of the condensation temperature T_c given by Eq. (8) with $b_0^* \simeq 0.0002$, $b_1^* \simeq -3.4$ and $b_2^* \simeq 47$.

It would be interesting to put the predicted universal relation (12) to further experimental test in order to find out whether it can also explain the values of b_2^* for other resonances of ^{39}K as well as for other atomic condensates by repeating the measurements of the second-order interactions effects performed in Ref. [36].

This work has been financially supported by the Brazilian agencies CNPq and CAPES. MdeLl thanks PAPIIT-UNAM for grant IN-100314 and MG for grant IN-116914.

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